

CRITICAL MATHEMATICS EDUCATION: AN APPLICATION OF PAULO FREIRE'S EPISTEMOLOGY

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Paulo Freire's critical education theory is "re-invented" in the context of a mathematics curriculum for urban working-class adults. The problems Freire poses for teachers in that context are explored, and work of other theorists which deepens or questions aspects of Freire's theory is discussed. Next, Freire's theory is applied to teaching basic mathematics and statistics for the social sciences. It is argued that such mathematical literacy is vital in the struggle for liberatory social change in our advanced technological society. Finally, this reflection on practice is used to pose further problems to be explored in the creation and re-creation of the "pedagogy of the oppressed."

Knowledge of basic mathematics and statistics is an important part of gaining real popular, democratic control over the economic, political, and social structures of our society. Liberatory social change requires an understanding of the technical knowledge that is too often used to obscure economic and social realities. When we develop specific strategies for an emancipatory education, it is vital that we include such mathematical literacy. Statistics is usually abandoned to "experts" because it is thought too difficult for most people to understand. Since this knowledge is also considered value-free, it is rarely questioned. In attempting to create an approach to mathematics education that can lead both to greater control over knowledge and to critical consciousness, it is important to have an adequate pedagogical theory that can guide and illuminate specific classroom practices. I want to argue that Paulo Freire's "pedagogy of the oppressed" can provide the theoretical foundation for that practice.

Freire's educational theory is complex. In this article I will focus on the problems he poses that are particularly pressing for teachers in schools in the United States. For this reason, I will not treat his theory on why revolutionary party leaders must also be educators, or his assumptions (historically grounded in the reality of the various Third World countries in which he has practiced) that these leaders would come from the bourgeoisie, "committing suicide as a class in order to rise again as revolutionary workers" (Freire, 1978, p. 16). Instead, I want to investi-

gate his epistemology, his theory about the relationship between education and social change, and his methodology for developing critical consciousness. Because of Freire's argument that critical education involves problem posing in which all involved are challenged to reconsider and re-create their prior knowledge, this presentation should be seen as an exploration intended to help extend our thinking, not as "Freire's definitive formulas-for-liberation." A discussion of my own experience teaching urban working-class adults¹ basic mathematics and statistics for the social sciences demonstrates ways in which Freire's theory can illuminate specific problems and solutions in critical teaching, and ways in which mathematics education can contribute to liberatory social change.

The Problems Freire Poses To Teachers In The United States

What is Knowledge?

Freire's epistemology is in direct opposition to the positivist paradigm currently dominant in educational theory. Positivists view knowledge as neutral, value-free, and objective, existing totally outside of human consciousness. Further, knowledge is completely separate from how people use it. Learning is the discovery of these static facts and their subsequent description and classification (Bredo & Feinberg, 1982). Giroux's (1981) critique of positivism in education theory focuses attention on what is omitted from that paradigm.

Questions concerning the social construction of knowledge and the constitutive interests behind the selection, organization, and evaluation of "brute facts" are buried under the assumption that knowledge is objective and value free. Information or "data" taken from the subjective world of intuition, insight, philosophy and nonscientific theoretical frameworks is not acknowledged as being relevant. Values, then, appear as the nemeses of "facts," and are viewed at best, as interesting, and at worst, as irrational and subjective emotional responses. (pp. 43-44)

Paulo Freire insists that knowledge is not static: that there is no dichotomy between objectivity and subjectivity, or between reflection and action: and that knowledge is not neutral.

For Freire, knowledge is continually created and re-created as people reflect and act on the world. Knowledge, therefore, is not fixed permanently in the abstract properties of objects, but is a process where gaining existing knowledge and producing new knowledge are "two moments in the same cycle" (Freire, 1982). In addition, knowledge requires subjects; objects to be known are necessary, but they are not sufficient.

Knowledge...necessitates the curious presence of subjects confronted with the world. It requires their transforming action on reality. It demands a constant searching....In the learning process the only person who really *learns* is s/he who...re-invents that learning. (Freire, 1973, p. 101)

Knowledge does not exist apart from human consciousness; it is produced by us collectively searching and trying to make sense of our world.²

So, for Freire, the world is "giving" rather than "given" (Collins, 1977, p. 82), and subjectivity and objectivity are not separate ways of knowing.

To deny the importance of subjectivity in the process of transforming the world and history is...to admit the impossible: a world without people.... On the other hand, the denial of objectivity in analysis or action... postulates people without a world...[and] denies action itself by denying objective reality. (Freire, 1970a, pp. 35-36)³

Because of the unity between subjectivity and objectivity, people cannot *completely* know particular aspects of the world—no knowledge is "finished." As humans change, so does the knowledge they produce. But, through constant searching and dialogue, we can continually refine our understanding in the sense that we can act more effectively.

This action and the reflection upon it that leads to new action are not separate moments of knowing. Reflection which is not ultimately accompanied by action to transform the world is meaningless, alienating rhetoric.⁴ Action which is not critically analyzed cannot sustain progressive change. Without reflection, people cannot learn from each other's successes and mistakes; particular activities need to be evaluated in relationship to larger collective goals. Only through praxis—reflection and action dialectically interacting to re-create reality—can people become subjects in control of organizing their society. Moreover, this praxis is not neutral. Knowledge does not exist apart from how and why it is used, in whose interest. Even, for example, in the supposedly neutral technical knowledge of how to cultivate potatoes, Freire asserts that

there is something which goes beyond the agricultural aspects of cultivating potatoes.... We have not only...the methods of planting, but also the question which has to do with the role of those who plant potatoes in the process of producing, for what we plant potatoes, in favor of whom. And something more. It is very important for the peasants...to think about the very process of work—what does working mean? (Brown, 1978, p. 63, note 1)

For Freire, the purpose of knowledge is for people to humanize themselves by overcoming dehumanization through the resolution of the fundamental contradiction of our epoch: that between domination and liberation.

An additional concept that illuminates Freire's epistemology by helping unpack the objective and subjective forces that shape knowledge and the reflective and active moments in knowing is the dialectic. Giroux (1981) defines this concept as

a critical mode of reasoning and behavior...[that] functions so as to help people analyze the world in which they live, to become aware of the constraints that prevent them from changing that world, and, finally, to help them collectively struggle to transform that world. (pp. 114, 116)

The central categories of Giroux's formulation of the dialectic—totality, mediation, appropriation, and transcendence—detail the various dimensions of a Freirean critical knowledge of reality. Totality involves understanding any fact or situation in its historical, socio-economic, political, and cultural context. So as we come to know a particular aspect of the world, we must be concerned with its causal relationships, with its connections to other phenomena, with who benefits from its continuance, and with how it relates to our humanization or dehumanization. As we explore these questions, the answers we formulate are mediated by the institutional structures of society, by our individual and class histories, by our depth psychology, by our current relationships, and by the specific details of the concrete moment in which we are involved. The category of mediation challenges the "taken-for-granted" by helping us unravel the layers of objective and subjective forces through which we make meaning. The category of appropriation focuses our attention on human agency—on how people's actions both continue and challenge the relations of domination which mark our society. Therefore, critical knowledge involves uncovering the limits and the possibilities of our actions for transforming the world. Finally, transcendence unites commitment with theory, insisting that we refuse to accept domination as a "fact" of existence and that we use our knowledge of the world to reconstruct society so that it is "free of alienating and oppressive social institutions and life forms" (Giroux, 1981, p. 122). Thus, the dialectic as a mode of analysis not only clarifies the critical nature of knowledge, but also points to the connections between critical knowledge and emancipatory social change.

Education and Liberatory Social Change

Although Freire insists that "There is no such thing as absolute ignorance or absolute wisdom" (1973, p. 43), he also maintains that in an oppressive society people's knowledge is at different levels. People with the most dominated, "semi-intransitive" consciousness have a fragmented, localized awareness of their situation and are unable to think

dialectically about it. Therefore, they view their condition as caused by their own failure and/or by "God." People living in more open societies naturally develop "naive transitive" consciousness where they begin to see causes in a broader context, but are still convinced that "causality is a static, established fact" (Freire, 1973, p. 44) and, therefore, not susceptible to change through their actions.

One of the major obstacles that the "pedagogy of the oppressed" must overcome is the participation of the oppressed in their own domination. Freire explores the structural, emotional, and cognitive factors behind this "culture of silence." In Brazil, the people had internalized their lack of participatory democratic experience under Portuguese imperialism. This emotional identification was strengthened by the myths the oppressors created that the status-quo represented the only possible situation because the oppressed were completely ignorant and powerless, while the rulers were omniscient and omnipotent. In such situations, the oppressed tend to fatalistically adjust to their condition. Since the relationships they have experienced and internalized involve the oppressor-oppressed division, their visions for a better life were very individualistic and focused on joining the oppressors rather than eliminating them.

However, as Freire insists, "the concept of semi-intransitivity does not signify the closure of people within themselves, crushed by an all-powerful time and space. Whatever their state, people are open beings" (1973, p. 17). One very important aspect of this hope for Freire is people's *conscientização*—their development of critical consciousness—which he maintains can only emerge through dialogical, problem-posing education. Since action cannot be dichotomized from reflection, and critical education develops critical knowledge, Freire views education as vital in helping people to become subjects involved in liberatory social change.⁴

An analysis of how education in the United States can lead to people's *conscientização* involves a focus on overcoming what Freire has called "massified," as opposed to semi-intransitive, consciousness. People with massified consciousness understand that humans change and control the world. But they believe that each individual acts from rational free choice rather than from a complex interplay of choice and manipulation. In *Cultural Action for Freedom* Freire begins an analysis of how the massified consciousness typical of advanced technological societies becomes the major factor in people's participation in their own domination:

The rationality basic to science and technology disappears under the extraordinary effects of technology itself, and its place is taken by myth-making irrationalism. . . . People begin thinking and acting according to the prescriptions they receive daily from the communications media rather than in response to their dialectical relationships with the world. In mass societies,

where everything is prefabricated and behavior is almost automatized, people are lost because they don't have to "risk themselves."...Technology... becomes...a species of new divinity to which [people] create a cult of worship (1970b, pp. 49-50).

Both the (apparent) complexities of technology and the (superficially) wonderful concrete changes it has made in daily life, from washing machines to word processors, convince people that control over our high-tech society must be left to "experts." Critical education in the United States, therefore, must counter this belief by showing people that they can understand how technology works, and in whose interest. Also, critical education must challenge and expose the contradictions in this society's definition of "progress" and "the good life."

The meaning of "massification" in highly industrialized societies is illuminated by the concepts of ideology and hegemony. These concepts can sharpen the analysis of how a massified consciousness is developed and perpetuated and point to ways in which education could help break it open. Kellner (1978), drawing on the work of Karl Korsch and Antonio Gramsci, develops a theory of ideology-as-hegemony and of "ideological regions," which demonstrates how ideology contains "anti-capitalist and oppositional moments—contradictions that produce space for ideological struggle and social change" (p. 59). For Kellner, ideological knowledge, in contrast to critical thought,

tends to suppress reflection, and resists changing its core ideas in the light of recalcitrant experience....Nonideological thought and discourse exercises consistent and systematic reflection and critique on its methods, presuppositions, doctrines, and goals. It continually tests its ideas in practice, remaining open to experience, flexible, and capable of critique, self-critique, and revision (p. 54).

The ideas and images about "the way the world is" that constitute an ideology become hegemonic when they serve to preserve the status quo, presenting it as "natural, good, and just" (p. 50). Hegemonic ideologies, however, are not simply *imposed* by the ruling classes and believed by the "duped" masses—these ideologies are *constructed* through negotiation so as to incorporate people's ideas in such a way that they are not dangerous to the ruling classes. This process leaves hegemonic ideology with contradictions and open to challenge.

In order to focus on these contradictions and challenges, Kellner refines his theory to detail various "ideological regions"—economic, political, social, cultural—"which reproduce in thought the practices, institutions, and relations in each realm of existence so as to legitimate it and achieve hegemony" (p. 58). Tensions among ideologies in different realms (e.g., the hedonistic consumer ethic vs. monogamy and the

family), contradictions between hegemonic ideology and reality (e.g., the ideological notion of equality vs. institutional racism), the fact that there is no *one* unifying hegemonic ideology—these all help to create an opening for education to develop critical theory which can in turn foster liberatory social change.

*Content and Methods for Education
for Critical Consciousness*

In developing a critical pedagogy we must consider both content and methods. Emancipatory content presented in a nonliberatory way reduces critical insights to empty words which cannot challenge students' taken-for-granted reality and cannot inspire commitment to radical change. Humanistic methods without critical content can make students "feel good," but cannot help them become subjects capable of using critical knowledge to transform their world.

Freire is adamant that the content of an education for critical consciousness must be developed by searching with the students for the ideas and experiences which give meaning to their lives (1970a, p. 118). These "generative themes" should be organized and "re-presented" dialectically so that the links between them, their relationship to the totality of ideas, hopes, values, and challenges of the epoch, their historical context, their relationship to the community, and their *raison d'être*, are all clarified. Only as people come to know these themes critically, as they realize how these themes support or contradict the dominant ideologies, do they see that "dehumanization, although a concrete historical fact, is *not* a given destiny but the result of an unjust order" (Freire, 1970a, p. 28). And only then are they motivated to intervene to transform that order.

Literacy becomes an important part of a liberatory curriculum because reading enables people to gain distance from the concrete immediacies of their everyday lives in order to understand more clearly how their lives are shaped by and in turn can shape the world (Freire, 1983, p. 11). Further the study of language is vital because

the object of the investigation [of generative themes] is not people (as if they were anatomical fragments), but rather the thought-language with which people refer to reality, the levels at which they perceive that reality, and their view of the world, in which their generative themes are found. (Freire, 1970a, p. 86)

Apple's (1979) analysis of labeling points to the value, in this context, of studying the language used to discuss the condition of the oppressed. He

gles for social change. In order to develop a "pedagogy of the oppressed," Freire contends that we need to explore the nonpositivist nature of the knowledge we are teaching, and the ways in which producing such knowledge deepens commitment and involves action to transform the world. The next section of this paper relates my experiences using Freire's theory to teach mathematics. The specific details are presented to provide a case study of how Freire's theory can inform critical teaching. They also support the belief that critical knowledge of statistics is vital to transforming our massified technological society.

Freire's Theory for Mathematics Teachers

All people reflect on their practice to some degree; mental and manual labor can never be completely divided. Even mathematics teachers who have never heard of Polya (1957, 1981) or Freire will think about problems such as how to explain the "sampling distribution of the mean" so that students do not confuse it with a distribution of scores within one sample. However, studying theory deepens the nature of these reflections; in particular, I believe that a theoretical framework changes the depth and types of questions one considers when thinking about one's practice. Freire's theory compels mathematics teachers to probe the nonpositivist meaning of mathematical knowledge, the importance of quantitative reasoning in the development of critical consciousness, the ways in which math anxiety helps sustain hegemonic ideologies, and the connections between our specific curriculum and the development of critical consciousness. In addition, his theory can strengthen our energy in the struggle for humanization by focusing our attention on the interrelationships between our concrete daily teaching practice and the broader ideological and structural context.

Freire's Epistemology and the Meaning of Basic Mathematics and Statistics Knowledge⁶

The mass media, most academic social scientists, and "common sense" assume that mathematical knowledge consists of neutral facts discovered, not created, by people through their interactions with the world. Cynics claim statistics are all self-serving lies. A Freirean analysis, different from both of these approaches, directs our reflections to the relationship between subjectivity and objectivity in producing mathematical knowledge.

A course such as "Statistics for the Social Sciences" affords many opportunities for examining how subjective choice is involved in describing

and collecting data, and in making inferences about the world. For example, Max (1981) and Greenwood (1981) show how the government makes military spending appear smaller by including funds held "in trust," such as Social Security, in the portion of the federal budget going for social services; and, counting war-related expenditures such as the production of new nuclear warheads, the space program, and veteran's programs, as part of various nonmilitary categories like the Department of Energy budget (the warheads!) and Direct Benefit Payments (veterans' income). The government calculates that 25% of the budget goes for "National Defense"; Max's and Greenwood's calculation give a figure of 57% of the budget going to pay for "Past, Present and Future Wars." Atkins and Jarrett (1979) show how significance tests, one of the most commonly used techniques in inferential statistics, can be used to "provide definite and apparently objective decisions, in a basically superficial way" (p. 103). One reason this occurs is because a 'favorable' numerical result in a significance test gives no assurance that the measurements used in the study are *meaningful*. In 1925, for example, Karl Pearson, an important figure in the development of modern statistical theory, found "statistically significant" differences in Jewish children's physical characteristics and intelligence—leading him to conclude they should not be allowed to immigrate into Great Britain (pp. 101–102). But, what *substantive* significance does this have when these characteristics are so clearly environmental? Also, the nature of probability *requires* statistical inference to be uncertain—a research hypothesis tested "significant at the .05 level" gives the impression of certainty, whereas it means there is a 5% chance that the hypothesis is false. Events with low probabilities sometimes do occur; significance tests only allow researchers to be reasonably (say, 95%) certain that the event described by their hypothesis is *not due to chance*. Moreover, the tests cannot determine which of many possible theories explains the event. For example, R.A. Fischer, author of a widely used modern statistics text, uses results from a chi-square test, showing a statistically significant greater frequency of criminality among monozygotic than among dizygotic twins of criminals, to conclude that this happens because of genetic factors. He ignores any other possible explanation, such as people's treatment and expectations of identical versus similar-looking twins (Schwartz, 1977, p. 28).

Freire's concept of critical knowledge further directs us to explore not merely how statistics are non-neutral, but why, and in whose interest. It is certainly not accidental that official statistics are much more useful to conservatives than to radicals. Nor is it accidental that, in spite of the technical weaknesses of significance tests, many standard social science computer packages lack convenient procedures for estimation, an alternative to significance tests that can be evaluated by statistical and *other* criteria and can facilitate comparison among investigations.

On the other hand, the thousands of government workers and university social scientists who produce this statistical knowledge are not *forced* to use methods whose outcome will uniformly support the ruling classes. An examination of the history of statistics can help explain how statistical knowledge "naturally" arises from the conditions of our society in such a way that its production is controlled by the ruling classes. Shaw and Miles (1979) trace its development to the expansion of commerce and the changing needs of the state. In 16th-century London, the crowded conditions of towns, which arose from the growth of markets, created the climate for widespread epidemics which led to the first collecting of mortality statistics. As these statistics were refined, they became more useful to the ruling classes. For example, William and Mary's government paid for loans to conduct the war against France with life annuities whose value was calculated using statistics on life expectancies of people in various age groups. In the 19th century, the rise of industrial capitalism led to the state's assuming a large role in providing conditions under which private industry could thrive, including the expansion and centralization of statistical knowledge. One consequence of this was that in 1832, the Statistical Department of the Board of Trade was charged with gathering and organizing material concerning British "wealth, commerce and industry."

Giroux's (1981) category of mediation extends this historical analysis by calling our attention to the combination of structural and individual factors which inform the production of this knowledge. One factor involves organizational "efficiency," which results in certain statistics being produced as by-products of administrative systems existing mainly for other purposes. For example, in England unemployment statistics are based on records kept by employment exchanges, so the workers who fail to register are omitted from the official reports (Hyman and Price, 1979). Another factor involves pressures on social scientists from journals that only accept articles which report statistically significant results, and from universities which grant tenure only to widely published professors. This "naturally" results in an underreporting of results that are *not* statistically-significant. Thus, one researcher by *chance* may produce and publish a statistically significant finding, while many others researching the same problem find no statistical significance, but since their work is not published, no conflicts among results can be detected (Atkins & Jarrett, 1979). Next, Giroux's category of appropriation focuses attention on how, in spite of the many factors resulting in what he calls a "selective affinity" for people to produce statistical knowledge to support the interests of the ruling classes, people can still learn from statistics. This is possible because statistical knowledge can be analyzed critically by examining its underlying interests and methods of collection, description, and inference, and by considering historical, philosophical, and

other theoretical insights along with the statistical knowledge. Finally, Giroux's category of transcendence insists that we not only criticize existing statistics, but that we also explore what new knowledge might be produced consistent with humanization. Along this line, Griffiths, Irvine, and Miles (1979) suggest that new statistical techniques for collecting data can be developed. For instance, interactive surveys could, instead of treating the respondents as isolated, passive objects, make them participants in analyzing how they can use the information gathered to improve their lives. Further, Shaw and Miles (1979) hypothesize that in a liberatory society

we would replace accountancy in terms of money and profit by accountancy in terms of social needs. We would replace the definition of social goals by those at the tops of the bureaucratic pyramids, by democratic self-control over all collective activities. We would then require new ways of measuring our needs and goals, which expressed their great variety rather than reduced them to money values or standards imposed from above. (p. 36)

Mathematics Education and Liberatory Social Change

Applying Freire's theory to mathematics education directs our attention to how most current uses of mathematics support hegemonic ideologies, how mathematics education also reinforces hegemonic ideologies, and how critical mathematics education can develop critical understanding and lead to critical action.

A significant factor in the acceptance of this society's hegemonic ideologies is that people do not probe the mathematical mystifications that in advanced industrial society function as vital supports of these ideologies. A mathematically illiterate populace can be convinced, for example, that social welfare programs are responsible for their declining standard of living, because they will not research the numbers to uncover that "welfare" to the rich dwarfs any meager subsidies given to the poor. For example, in 1975 the maximum payment to an Aid for Dependent Children family of four was \$5,000 and the average tax loophole for each of the richest 160,000 taxpayers was \$45,000 (Babson & Brigham, 1978, p. 37). Also in 1980, \$510 million of our tax money paid for new airports so that private pilots would not land their planes at large commercial airports (Judis & Moberg, 1981, p. 22.) Further, people's misconception that statistical knowledge is objective and value-free closes off challenges to such data. As Marcuse (1964) argues,

In this society, the rational rather than the irrational becomes the most effective vehicle of mystification....For example, the scientific approach to the vexing problem of mutual annihilation—the mathematics and calcula-

tions of kill and over-kill, the measurement of spreading or not-quite-so-spreading fallout...—is mystifying to the extent to which it promotes (and even demands) behavior which accepts the insanity. It thus counteracts a truly rational behavior—namely, the refusal to go along, and the effort to do away with the conditions which produce the insanity. (pp. 189-190)

Traditional mathematics education supports the hegemonic ideologies of society, especially through what Giroux calls "structured silences." Even trivial math applications like totaling grocery bills carry the ideological message that paying for food is natural and that society can only be organized in such a way that people buy food from grocery stores. Also it is rare that students are asked to evaluate their own understanding of math. My students are convinced that they are cheating if they check their own work using an answer key or with other people, and they have no experience analyzing which specific topics are giving them difficulty. In the past, when they could not do an assignment, they just expressed general confusion and gave control of their learning to the teacher to "diagnose" what they needed to review. This reinforces the hegemonic ideology of "expertise"—that some people have (i.e., own) a great deal of knowledge which can only be obtained from them and which they will impart *only if* you "follow the rules."

One of the obstacles that critical mathematics education must overcome in the United States is people's math "anxiety." Since, as Freire, stresses, people who are not aware of the *raison d'être* of their situation, fatalistically "accept" their exploitation, teachers and students must consider the causes behind math "anxiety" as part of developing critical mathematics education. The immediate pedagogical causes of the situation—such as meaningless rote drill, taught so that it requires extensive memorization, and unmotivated applications which are unrelated to the math one actually uses in everyday life—create a situation where people "naturally" avoid mathematics (Hilton, 1980). Discussions with students helped me to re-conceptualize these pedagogical causes in terms of misconceptions about learning. One misconception concerns the group process in learning. Students often feel they must be able to solve a problem on their own before they can contribute to the group. They do not realize that collectively a group can solve problems that individual members working alone could not solve. Another misconception is the idea that 'a wrong answer is totally wrong, nothing can be learned from analyzing it' (Frankenstein, 1983).

Understanding the deeper causes of math "anxiety" involves an examination of how the structures and hegemonic ideologies of our society result in different groups being more affected than others by this "anxiety." It also involves recognition that, to some extent, people participate in their own mathematical disempowerment. Considerable research (summarized in Beckwith, 1983) has documented that

sex differences in mathematical training and attitudes... are *not* the result of free and informed choice.... They are the result of many subtle (and not so subtle) forces, restrictions, stereotypes, sex roles, parental-teacher-peer group attitudes, and other cultural and psychological constraints. (Ernest et al., 1976, p. 611).

Further research needs to be done. In particular we need to investigate how differential treatment based on race and class interacts with mathematics "anxiety" and avoidance. We also need to explore why the research on math anxiety has focused only on the relationship between sex and mathematics learning.

In addition to the effects of sexism, racism, and classism, the hegemonic ideology of "aptitudes"—the belief, in relationship to mathematics, that only some people have a "mathematical mind"—needs to be analyzed. Women's belief that men have more "mathematical aptitude" has been explored. Tobias (1978) discusses research investigating the hidden messages in math textbook content and images; Beckwith (1983) summarizes studies of media influence on children's perceptions of boys' allegedly superior math abilities. However, Apple's (1979) discussion of labeling suggests that more research needs to be done on the contradictory effects of the term "math anxiety." Students are initially relieved that their feelings about mathematics are so common that educators "have a name for them." But in fact this label focuses the problem, and therefore the solutions, on individual failure rather than on the broader societal context which plays such a significant role in producing personal "math anxiety." Bisseret (1979) demonstrates how language functions ideologically to support the belief that "a difference in essence among human beings...predetermines the diversity of psychic and mental phenomena" (p. 2). Her analysis illuminates the role which this ideology of aptitudes plays in people's beliefs that the given structure of society is "natural" and "inevitable," and suggests further research to be done in uncovering the complex factors behind the ideology of a "mathematical mind." Bisseret argues that this ideology results in class-specific language; we need to consider how this language encourages dominated groups to believe and act as if they have "nonmathematical minds."

Critical mathematics education can challenge students to question these hegemonic ideologies by using statistics to reveal the contradictions (and lies) underneath the surface of these ideologies by providing learning experiences where students and teachers are "co-investigators" and where math "anxious" students overcome their fears. Further, critical mathematics education can link this questioning with action, both by illustrating how organized groups of people are using statistics in their struggles for social change and by providing information on such local groups as students may wish to join. Above all, critical mathematics education must take seriously Marcuse's (1964) injunction that

The trouble is that the statistics, measurements, and field studies of empirical sociology and political science are not rational enough. They become mystifying to the extent to which they are isolated from the truly concrete context which makes the facts and determines their function. This context is larger and other than that of the plants and shops investigated, of the towns and cities studied, of the areas and groups whose public opinion is polled or whose chance of survival is calculated....This real context in which the particular subjects obtain their real significance is definable only within a *theory* of society. (p. 190)

Content and Methods in Critical Mathematics Education

In order to apply Freire's theory to critical mathematics education we need to consider what mathematics knowledge is implied by, and would clarify, our students' "generative themes." In most school settings, teachers cannot get to know their students as well as Freire's teams got to know the communities in which they taught. However, teachers can ask students about the issues that concern them at work, about the nonwork activities that interest them, about topics they would like to know in more depth, and so forth. These discussions can indicate the starting point for the curriculum. Then the teacher's contribution can be to link up the students' issues with an investigation of the related hegemonic ideologies. Any topic can be so connected; for example, art can lead to an exploration of such areas as the ideology of "high status" knowledge, the ideology of "taste," and the commodification of culture.⁷

In addition, almost all of the basic math and statistics skills and concepts, as well as the critical nature of statistical knowledge, can be learned in the context of working on applications which challenge the contradictions involved in supporting hegemonic ideologies.⁸ For example Max and Greenwood's critique of the official statistics on the military portion of the federal budget can be used to learn percents and circle graphs. In addition, students can discuss how they would decide to present the critique, and what aspects of this research and presentation they control. Would they choose to present their critique using raw data, percents, or graphs? Do they agree with Max that the space program should be considered part of the cost of "Past, Present and Future Wars"? Discussing how to present the statistics to demonstrate that the United States is a welfare state for the rich can include practice of arithmetical operations; students need to divide in order to describe the tax loophole data as "each of the richest 160,000 taxpayers got nine times as much money as the maximum AFDC grant for a family of four." This same data helps students learn about the meaning of large numbers; they can consider the services that the total taxes not paid by these rich 160,000

(\$7,200,000,000 = \$7.2 billion) could have provided if this money were included in the federal budget. For a final example, Gray (1983) presents positive uses of statistical techniques (such as chi-square and regression analysis) in legal cases. In one situation, such techniques were used to show that in jury selection "a hypothesis of random selection, that is, of no discrimination, is so improbable as to make it likely that some other process must have been at work" (p. 72).

Not only can math skills and concepts be learned in the classroom from applications which challenge the hegemonic ideologies, but interested students can also work with the many groups uniting reflection about statistics with action for social change. The Coalition for Basic Human Needs, in Boston, uses statistics, (for example those showing that actual shelter costs in every major Massachusetts city exceed the AFDC welfare grant) to fight for decent conditions for (poor) welfare recipients. The International Association of Machinists had a statistician prepare a report on "The Impact of Military Spending on the Machinists Union" (Anderson, 1979, note 3) which documents that "as the military budget goes up, and procurement contracts rise, Machinists jobs in military industry steadily decline" (p. 1). Counter-Information Services (CIS), a London-based group of journalists, trade unionists, and statisticians, reconceptualizes information in official corporate reports, at the request of workers at the companies involved. CIS issues "Anti-Reports" which present a critical analysis of the company's statistics. CIS's Anti-Report on Ford (1978, note 4), for example, used that company's data to show that Ford had been exaggerating the profitability of its West German operation and understating that of its British plants. Since the United Kingdom workers were more militant in their demands than the German workers, Ford used its doctored statistics to threaten the UK workers with their alleged poor performance. For another example, CIS's Anti-Report on Rio Tinto Zinc (RTZ) Corporation (1972, note 5), used RTZ's data that 42% of its profits were made in South Africa, whereas only 7.7% of its assets were located there, along with additional information CIS researched, to support its charge that these high profits came directly from the low wages paid to RTZ's black miners.

As these math examples challenge students to reconsider their previously "taken-for-granted" beliefs, they also deepen and increase the range of questions they ask about the world. Once the idea of comparing the results of military vs. civilian spending on jobs is introduced, one can then ask that same question of other government spending. For example, are more jobs created through spending on energy conservation or nuclear power? Further, by learning and re-creating a theory of math education and social change with their teachers, students can develop their ability to critique ideology in general.

Freire's methodology shares much in common with humanistic ideas on student-centered teaching but his ideas go beyond those methods in terms of their *intent*. They are not merely the techniques that any dedicated teacher who respected his or her students might use. Instead, they are intended to be part of the process of developing new social relations in the struggle for humanization. Freire's methodology directs math teachers' attention to how students with large gaps in their mathematical background can in practice co-investigate the statistical aspects of their "generative themes." It also directs teachers to consider how students can become independent at decoding the problems coded in the barrage of quantitative data encountered in their lives.

By exploring the statistical aspects of students' themes in such a way that the mathematics involved starts at a very basic level, and by having students pose problems about the data even if they cannot yet solve those problems, teachers and students are *truly* co-researchers. Since math teachers will probably not have previously investigated many of the suggested themes, students are likely to ask questions that teachers will not be able to answer and which students and teachers will have to research together. For example, the following chart can be used to start a dialogue with students who have previously suggested the theme of racism:

Table 1

Median Income of Black and White Families 1969-1977

	1969	1972	1974	1975	1976	1977
Black	\$5,999	6,864	8,006	8,779	9,242	9,563
White	9,794	11,549	13,408	14,268	15,537	16,740

(Source: Census Bureau, Current Population Reports, P-60 Series)

Students can initially be asked to describe what the main point of the chart is—an exercise in which they can practice such skills as comparing numbers, subtraction, or finding what percent one number is of another number. As the investigation deepens, students and teachers are equals in problematizing what other statistics would clarify the theme of racism (e.g., comparisons by race of maternal mortality rates; comparisons by race of unemployment statistics; comparisons of latinos with blacks and whites). The importance of statistics in revealing institutional patterns, in contrast with personal instances of racism, is also brought out by this research. Further depth is added to the investigation by students and teachers jointly finding and considering various social science studies which use more advanced statistical techniques to clarify the theme. Reich (1978), for example, uses correlation coefficients between various

statistical measures of racism and white incomes to show that racism results in lower wages for white, as well as black, workers and higher profits for the capitalist class. Finally, any thematic investigation must include more than just statistical data. As Reich comments, in this case

the simple economics of racism does not explain why many workers seem to be so vehemently racist, when racism is not in their economic self-interest. In noneconomic ways, racism helps to legitimize inequality, alienation, and powerlessness—legitimization that is necessary for the stability of the capitalist system as a whole... Through racism, poor whites come to believe that their poverty is caused by blacks who are willing to take away their jobs, and at lower wages, thus concealing the fact that a substantial amount of income inequality is inevitable in a capitalist society. (p. 387)

The above example also illustrates how a dialogical analysis involving the interpretation of statistical data helps students practice the slow, careful thinking necessary to produce any critical knowledge. This practice, combined with opportunities to reflect on the learning process, helps students to become independent learners. Many such opportunities come from involving students in evaluation. For example, as students work on review problems they can answer keys which pose questions about potential errors. Thereby, students are encouraged to pinpoint their own misunderstandings and determine how well they understood each problem. They can be asked to choose between "wrong answer because confused about _____"; "correct answer but unsure of method"; and "understand well enough to teach others." Students can also learn a lot about posing problems by evaluating the clarity, the difficulty, and the interest, of other students' and teachers' problems. Finally, having students keep a math journal is another method of having them reflect about their learning process. Journals can be vents for students' feelings about math and can act as a concrete record of progress for students who too often belittle their successes and focus on what they cannot do. The journal helps students realize that they can now accomplish what one month ago they thought was impossible. It helps them clarify which learning techniques worked best and why, and can give them personal feedback from the teacher and/or other students offering encouragement and alternative perspectives. The journal is also another way for students to be involved with the teacher in planning the curriculum, as their comments about their learning and their reactions to the class are considered in future lessons. Following is an example from one of my students' journals:

Class #6: I know that I ended my last entry into this journal as saying that; "I am ready to tackle the next class," but I wasn't. I was very tired and became bored at the very start of class. I have to learn to control my feelings

of being critical of other people problems in Algebra. I found myself thinking of the questions that some of the others asked as being elementary. I just assumed that if I understand, everyone should. Some of the problems I did have a little difficulty doing them, but I did not mention it in class because I felt that I would sound stupid or should I say unable to comprehend what was being said. Finally I began to fight the feelings that I had about other people problems and started being more attentive of what was being asked. I began to understand more and more and at one point, the questions that I wanted to ask were answered so, it wasn't so stupid after all.

My comments in the margin noted that it is hard to be patient with others' problems, but that after all, you want others to be patient with your problems. I suggested that it might be more interesting for her if she tried to answer the other students' problems, helping them to see exactly what was confusing to them. I also challenged her use of the label "stupid," and praised her insight into the learning process. I ended by asking if she would read this entry to the entire class as a way of introducing a discussion on what we can learn from collective work. This journal entry taught me about the importance of such discussions in helping everyone understand how much can be learned whenever anyone poses a problem.

Conclusion

The context in which we are working in the United States is quite different from the culture-circle context in and for which Freire developed his theory. In this article, I have attempted to convince people working in U.S. schools that Freire's theory contains many insights which we can use to inform our practice. Here I want to pose some problems arising from practice in our context that suggest areas of Freire's theory we need to develop further. These include the roles and responsibilities of students, the pressures on teachers, the complexities of moving students from massified to critical consciousness, and the tenuousness of the link between an emerging critical consciousness and radical social change.

Freire focuses on the responsibilities of teachers to challenge students' taken-for-granted beliefs, while simultaneously insuring that students become their "co-investigators" in this process. What responsibilities, then, do students have? How do we deal with the daily concrete reality of adult students whose work and family commitments make it difficult for them to do their "homework" or even attend class? How do we work within the enormous tensions created in our society between students' desire for individual "advancement" and our radical vision of collective progress?

Teachers, of course, are also affected by the pressures of daily life and the structures of our workplace. Freedman, Jackson, and Boles (1983) have shown how the conditions that elementary school teachers encounter in their day-to-day school situation—conditions such as the overwhelming emphasis on quantification (both in scoring children and in keeping records), the growing lack of control over curriculum (separating conception from execution), the isolation from their peers, the condescending treatment by administrators, and the massive layoffs of veteran teachers—"naturally" produce the frustration and anger that the mass media labels as "burn-out." In what struggles must we engage in order to change these conditions and sustain our energy to teach?

It is often tempting to abandon dialogical education, because of these pressures on students and teachers, because students have internalized misconceptions about learning and about their intellectual abilities from their previous schooling, and because we can get such quick positive feedback and (superficial) positive results by "banking" humanistically. But students' desire for "banking" education in an academic setting does not mean that they are not independent learners in many other situations. Freire discusses how in the transition from semi-intransitive consciousness, myths from the former stage remain even as the consciousness becomes more critical and open to new ideas (1970b). In addition to this overlapping of levels of consciousness, my practice calls attention to the nonlinear character of the levels of consciousness, and poses the problem of how to make a bridge from the critical insights my students have in some areas to their developing an overall critical approach to knowledge. My students' journals show how difficult it is for them to maintain a totalizing movement; entries show frequent "ups" and "downs" in self-image, and move between critical insight and myth. It seems clear that if the dialogical classroom experience is isolated and students are treated as objects in most other situations, then only fragments of critical consciousness can develop.

Further, these fragments are often theoretical, unconnected with practice. In both my experience and that of others (Rothenberg, 1983), the critical use of quantitative data can crack open hegemonic ideologies and students do become angry and intellectually committed to social change. But that does not necessarily mean they then join organizations working against oppression. Some even take jobs in business after getting their degree. Critical individual change does occur—when students overcome their math anxiety and learn math, they have a concrete, deep experience that "*things can change.*" They also develop the ability to critique and they increase their questioning of the conditions in which they live. It may be that the most critical collective change that a pedagogy of the op-

pressed can bring about in our circumstances is a subtle shift in climate which will aid the progress of liberatory social change.

Understanding the limits of our situation can increase our energy to focus on the radical possibilities of education as a force to promote emancipatory change. Using Paulo Freire's ideas as the theoretical foundation for our classroom practice situates that individual practice within the larger ideological and political struggle for humanization. We become more deeply committed as we realize how our actions are connected to this collective struggle. Using the term "militants" for people committed to justice and liberation (1978, p. 73), Freire argues that:

Militancy forces us to be more disciplined and to try harder to understand the reality that we, together with other militants, are trying to transform and re-create. We stand together alert against threats of all kinds. (1978, p. 146)

Footnotes

¹The students at my school are adults who have a clear commitment to work in public or community service. Their average age is 35, about 70% are women, and about 30% are people of color.

²Matthews (1981) traces Freire's emphasis on the social nature of thought to Karl Mannheim's philosophy that strictly speaking it is incorrect to say that individuals think; it is more correct to insist that they participate in thinking further what others have previously thought.

³One of Freire's first comments at the Boston College course he taught (July 1982) concerned his debt to the many American women who wrote to him praising *Pedagogy of the Oppressed* but criticizing his sexist language. He has changed his language; I therefore, change his quotes in this respect.

⁴Freire's writing on the details of how critical consciousness leads to radical social change (e.g., "This pedagogy makes oppression and its causes objects of reflection by the oppressed, and from that reflection will come their necessary engagement in the struggle for their liberation" (1970a, p. 33)) leaves him open to Mackie's critique that by ignoring "the political economy of revolution in favour of an emphasis on its cultural dimension... [Freire's] talk of revolution... tends to become utopian and idealized" (Mackie, 1981, p. 106). However, Freire's comments at his 1982 Boston College course (e.g., "in meetings like this we cannot change the world, but we can discover and we may become committed") convince me that he recognizes the limitations as well as the possibilities of education in bringing about liberatory social change. His writing, possibly, concentrates on the role of human consciousness in changing the world as a counter to overly determined structuralist theories of revolution.

⁵For a discussion of the specific conditions in Brazil under which Freire developed his theory and practice, see "Imperialism, Underdevelopment and Education" by Barnard (1981). For a detailed presentation of Freire's methodology, see Freire, 1973, pp. 41-84.

⁶Although this paper focuses on basic mathematics and statistics, Freire's theory can also illuminate other areas of mathematics knowledge. Some of these

connections are suggested by the ideas about the nature of abstract mathematical knowledge in Gordon (1978) and Kline (1980). In his introduction, Kline quotes Hermann Weyl, one of the most prominent mathematicians of the 20th century, " 'Mathematizing' may well be a creative activity of man, like language or music, of primary originality, whose historical decisions defy complete objective rationalization" (p. 6).

⁷Any topic can be connected to mathematics also: there are always statistics about that topic. In this case, there are even a number of contemporary artists whose work is based on specific mathematical structures (Frankenstein, 1982).

⁸For more basic mathematics examples, see my article which focuses on content and methods (Frankenstein, 1981). For more statistics examples, see Horwitz and Ferleger, 1980.

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